Test Paper : **III**

Test Subject : **MATHEMATICAL SCIENCE**

Test Subject Code : **K-2617**

Test Booklet Serial No. : ____________________

OMR Sheet No. : ____________________

Roll No. | | | | | | | | |

(Figures as per admission card)

Name & Signature of Invigilator/s

Signature : ____________________

Name : ____________________

Paper : III

Subject : MATHEMATICAL SCIENCE

Time : 2 Hours 30 Minutes

Maximum Marks : 150

Number of Pages in this Booklet : **16**

Number of Questions in this Booklet : **75**

ಅಭ್ಯರ್ಥಿಗಳಿಗೆ ಸೂಚನೆಗಳು

1. ಈ ಪುಟದ ಮೇಲ್ತುದಿಯಲ್ಲಿ ಒದಗಿಸಿದ ಸ್ಥಳದಲ್ಲಿ ನಿಮ್ಮ ರೋಲ್ ನಂಬರನ್ನು ಬರೆಯಿರಿ.
2. ಈ ಪತ್ರಿಕೆಯು ಬಹು ಆಯ್ಕೆ ವಿಧದ ಎಪ್ಪತ್ತೈದು ಪ್ರಶ್ನೆಗಳನ್ನು ಒಳಗೊಂಡಿದೆ.
3. ಪರೀಕ್ಷೆಯ ಪ್ರಾರಂಭದಲ್ಲಿ, ಪ್ರಶ್ನೆಪುಸ್ತಿಕೆಯನ್ನು ನಿಮಗೆ ನೀಡಲಾಗುವುದು. ಮೊದಲ 5 ನಿಮಿಷಗಳಲ್ಲಿ ನೀವು ಪುಸ್ತಿಕೆಯನ್ನು ತೆರೆಯಲು ಮತ್ತು ಕೆಳಗಿನಂತೆ ಕಡ್ಡಾಯವಾಗಿ ಪರೀಕ್ಷಿಸಲು ಕೋರಲಾಗಿದೆ.
   (i) ಪ್ರಶ್ನೆ ಪುಸ್ತಿಕೆಗೆ ಪ್ರವೇಶಾವಕಾಶ ಪಡೆಯಲು, ಈ ಹೊದಿಕೆ ಪುಟದ ಅಂಚಿನ ಮೇಲಿರುವ ಪೇಪರ್ ಸೀಲನ್ನು ಹರಿಯಿರಿ. ಸ್ಟಿಕ್ಕರ್ ಸೀಲ್ ಇಲ್ಲದ ಅಥವಾ ತೆರೆದ ಪುಸ್ತಿಕೆಯನ್ನು ಸ್ವೀಕರಿಸಬೇಡಿ.
   (ii) **ಪುಸ್ತಿಕೆಯಲ್ಲಿನ ಪ್ರಶ್ನೆಗಳ ಸಂಖ್ಯೆ ಮತ್ತು ಪುಟಗಳ ಸಂಖ್ಯೆಯನ್ನು ಮುಖಪುಟದ ಮೇಲೆ ಮುದ್ರಿಸಿದ ಮಾಹಿತಿಯೊಂದಿಗೆ ತಾಳೆ ನೋಡಿರಿ. ಪುಟಗಳು/ಪ್ರಶ್ನೆಗಳು ಕಾಣೆಯಾದ, ಅಥವಾ ದ್ವಿಪ್ರತಿ ಅಥವಾ ಅನುಕ್ರಮವಾಗಿಲ್ಲದ ಅಥವಾ ಇತರ ಯಾವುದೇ ವ್ಯತ್ಯಾಸದ ದೋಷಪೂರಿತ ಪುಸ್ತಿಕೆಯನ್ನು ಕೂಡಲೆ 5 ನಿಮಿಷದ ಅವಧಿ ಒಳಗೆ, ಸಂವೀಕ್ಷಕರಿಂದ ಸರಿ ಇರುವ ಪುಸ್ತಿಕೆಗೆ ಬದಲಾಯಿಸಿಕೊಳ್ಳಬೇಕು. ಆ ಬಳಿಕ ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯನ್ನು ಬದಲಾಯಿಸಲಾಗುವುದಿಲ್ಲ, ಯಾವುದೇ ಹೆಚ್ಚು ಸಮಯವನ್ನೂ ಕೊಡಲಾಗುವುದಿಲ್ಲ.**
4. ಪ್ರತಿಯೊಂದು ಪ್ರಶ್ನೆಗೂ (A), (B), (C) ಮತ್ತು (D) ಎಂದು ಗುರುತಿಸಿದ ನಾಲ್ಕು ಪರ್ಯಾಯ ಉತ್ತರಗಳಿವೆ. ನೀವು ಪ್ರಶ್ನೆಯ ಎದುರು ಸರಿಯಾದ ಉತ್ತರದ ಮೇಲೆ, ಕೆಳಗೆ ಕಾಣಿಸಿದಂತೆ ಅಂಡಾಕೃತಿಯನ್ನು ಕಪ್ಪಾಗಿಸಬೇಕು.
   ಉದಾಹರಣೆ : (A) (B) ● (D)
   (C) ಸರಿಯಾದ ಉತ್ತರವಾಗಿದ್ದಾಗ.
5. ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಗಳನ್ನು, **ಪತ್ರಿಕೆ III ಪುಸ್ತಿಕೆಯೊಳಗೆ ಕೊಟ್ಟಿರುವ OMR ಉತ್ತರ ಹಾಳೆಯಲ್ಲಿ ಮಾತ್ರವೇ ಸೂಚಿಸತಕ್ಕದ್ದು.** OMR ಹಾಳೆಯಲ್ಲಿನ ಅಂಡಾಕೃತಿ ಹೊರತುಪಡಿಸಿ ಬೇರೆ ಯಾವುದೇ ಸ್ಥಳದಲ್ಲಿ ಗುರುತಿಸಿದರೆ, ಅದರ ಮೌಲ್ಯಮಾಪನ ಮಾಡಲಾಗುವುದಿಲ್ಲ.
6. OMR ಉತ್ತರ ಹಾಳೆಯಲ್ಲಿ ಕೊಟ್ಟ ಸೂಚನೆಗಳನ್ನು ಜಾಗರೂಕತೆಯಿಂದ ಓದಿರಿ.
7. ಎಲ್ಲಾ ಕರಡು ಕೆಲಸವನ್ನು ಪುಸ್ತಿಕೆಯ ಕೊನೆಯಲ್ಲಿ ಮಾಡತಕ್ಕದ್ದು.
8. ನಿಮ್ಮ ಗುರುತನ್ನು ಬಹಿರಂಗಪಡಿಸಬಹುದಾದ ನಿಮ್ಮ ಹೆಸರು ಅಥವಾ ಯಾವುದೇ ಚಿಹ್ನೆಯನ್ನು, ಸಂಗತವಾದ ಸ್ಥಳ ಹೊರತು ಪಡಿಸಿ, OMR ಉತ್ತರ ಹಾಳೆಯ ಯಾವುದೇ ಭಾಗದಲ್ಲಿ ಬರೆದರೆ, ನೀವು ಅನರ್ಹತೆಗೆ ಬಾಧ್ಯರಾಗಿರುತ್ತೀರಿ.
9. ಪರೀಕ್ಷೆಯು ಮುಗಿದನಂತರ, ಕಡ್ಡಾಯವಾಗಿ OMR ಉತ್ತರ ಹಾಳೆಯನ್ನು ಸಂವೀಕ್ಷಕರಿಗೆ ನೀವು ಹಿಂತಿರುಗಿಸಬೇಕು ಮತ್ತು ಪರೀಕ್ಷಾ ಕೊಠಡಿಯ ಹೊರಗೆ OMR ನ್ನು ನಿಮ್ಮೊಂದಿಗೆ ಕೊಂಡೊಯ್ಯಕೂಡದು.
10. ಪರೀಕ್ಷೆಯ ನಂತರ, ಪರೀಕ್ಷಾ ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯನ್ನು ಮತ್ತು ನಕಲು OMR ಉತ್ತರ ಹಾಳೆಯನ್ನು ನಿಮ್ಮೊಂದಿಗೆ ತೆಗೆದುಕೊಂಡು ಹೋಗಬಹುದು.
11. **ನೀಲಿ/ಕಪ್ಪು ಬಾಲ್‌ಪಾಯಿಂಟ್ ಪೆನ್ ಮಾತ್ರವೇ ಉಪಯೋಗಿಸಿರಿ.**
12. **ಕ್ಯಾಲ್ಕುಲೇಟರ್, ವಿದ್ಯುನ್ಮಾನ ಉಪಕರಣ ಅಥವಾ ಲಾಗ್ ಟೇಬಲ್ ಇತ್ಯಾದಿಯ ಉಪಯೋಗವನ್ನು ನಿಷೇಧಿಸಲಾಗಿದೆ.**
13. **ಸರಿ ಅಲ್ಲದ ಉತ್ತರಗಳಿಗೆ ಋಣ ಅಂಕ ಇರುವುದಿಲ್ಲ.**
14. **ಕನ್ನಡ ಮತ್ತು ಇಂಗ್ಲೀಷ್ ಆವೃತ್ತಿಗಳ ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಗಳಲ್ಲಿ ಯಾವುದೇ ರೀತಿಯ ವ್ಯತ್ಯಾಸಗಳು ಕಂಡುಬಂದಲ್ಲಿ, ಇಂಗ್ಲೀಷ್ ಆವೃತ್ತಿಗಳಲ್ಲಿರುವುದೇ ಅಂತಿಮವೆಂದು ಪರಿಗಣಿಸಬೇಕು.**

Instructions for the Candidates

1. Write your roll number in the space provided on the top of this page.
2. This paper consists of seventy five multiple-choice type of questions.
3. At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as below :
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of the cover page. Do not accept a booklet without sticker seal or open booklet.
   (ii) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.**
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : (A) (B) ● (D)
   where (C) is the correct response.
5. Your responses to the question of Paper III are to be indicated in the **OMR Sheet kept inside the Booklet**. If you mark at any place other than in the circles in OMR Sheet, it will not be evaluated.
6. Read the instructions given in OMR carefully.
7. Rough Work is to be done in the end of this booklet.
8. If you write your name or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
9. You have to return the test OMR Answer Sheet to the invigilators at the end of the examination compulsorily and must NOT carry it with you outside the Examination Hall.
10. You can take away question booklet and carbon copy of OMR Answer Sheet after the examination.
11. **Use only Blue/Black Ball point pen.**
12. **Use of any calculator, Electronic gadgets or log table etc., is prohibited.**
13. **There is no negative marks for incorrect answers.**
14. **In case of any discrepancy found in the Kannada translation of a question booklet the question in English version shall be taken as final.**

MATHEMATICAL SCIENCE

PAPER – III

Note : This paper contains **seventy-five (75)** objective type questions. **Each** question carries **two (2)** marks. **All** questions are **compulsory**.

1. The point on the plane $2x + 3y - z = 5$ which is nearest to the origin is

(A) $(0, 0, -5)$

(B) $(3, 1, 4)$

(C) $\left(\frac{5}{7}, \frac{15}{14}, \frac{-5}{14}\right)$

(D) $(2, 1, 2)$

2. Which one of the following statements is true ?

(A) $\mathbb{R}$ in the lower limit topology is second countable

(B) Every compact metrizable space is second countable

(C) Every second countable Hausdorff space is metrizable

(D) If X has the discrete topology, then X is not paracompact

3. Consider $f(x) = x^3 + 2x^2 - 3x - 1$, with a starting approximation of $x_0 = 1$. Then fourth iteration of Newton-Raphson method produces the root of f(x) as

(A) 1.198695

(B) 1.17981

(C) 1.20191

(D) 1.198691

4. The initial value problem $\frac{dy}{dx} = y^{1/3}$, $y(0) = 0$.

(A) Has a unique solution in $\mathbb{R}$

(B) Has no non-zero solution in $\mathbb{R}$

(C) Has no solution in $\mathbb{R}$

(D) Has more than one solution in $\mathbb{R}$

5. The complete general solution of

$$\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial^2 x \partial y} + 4\frac{\partial^3 z}{\partial y^3} = e^{x+2y} \text{ is}$$

(A) $z = f_1(y-x) + f_2(y+2x) + f_3(y^2+2x) + \frac{1}{27}e^{x+2y}$

(B) $z = f_1(y-x) + f_2(y+2x) + f_3(y^2+2x) + \frac{1}{27}e^{y-x}$

(C) $z = f_1(y-x) + f_2(y+2x) + xf_3(y+2x) + \frac{1}{27}e^{x+2y}$

(D) $z = f_1(y-x) + f_2(y+2x) + y f_3(y+2x) + \frac{1}{27}e^{y+2x}$

6. The general solution of the partial differential equation

$$\left(\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2\right)y=\left(\frac{\partial z}{\partial y}\right)z \text{ is}$$

(A) $(x+a)^2+(y+z)^2=b^2$, where a and b are arbitrary constants

(B) $(x + a)^2 + y^2 = bz^2$, where a and b are arbitrary constants

(C) $(x + y)^2 + az^2 = b^2$, where a and b are arbitrary constants

(D) $ax^2 + by^2 = (x + y)^2$, where a and b are arbitrary constants

7. Which one of the following is true on Voltera integral equations of the second kind ?

(A) Resolvent Kernel $H(x, y; \lambda)$ is given by a sum of iterated Kernels

(B) Resolvent Kernel $H(x, y; \lambda)$ is given by a product of iterated Kernels

(C) Resolvent Kernel $H(x, y; \lambda)$ is a separable Kernel

(D) Resolvent Kernel satisfies the integral equation $H(x, y; \lambda) = K(x, y) + \lambda\int_a^b K(x, t)\, H(x, y; \lambda)dt$

8. The extremal for the functional

$\int_a^b [12xy+(y')^2]\, dx$ is given by

(A) $y(x)=x^3+c_1x+c_2$, where c_1 and c_2 are real constants

(B) $y(x)=x^2+\sin x+c$, where c is a real constant

(C) $y(x)=2x^2+\cos x+e^x$

(D) $y(x)=0$

9. Under the transformation $w=\frac{z-i}{z+i}$, the real axis in the z-plane is mapped into the circle $|w| = 1$. Which portion of the z-plane corresponds to the interior of the circle $|w| = 1$ among the following ?

(A) Upper half plane

(B) Lower half plane

(C) Left half plane

(D) Right half plane

10. If $z\in\mathbb{C}$ and e^{z^2} is written as $e^{z^2}=u(x, y) + iv(x, y)$, then $v(x, y)$ is given by

(A) $e^{x^2-y^2}$. sin 2xy

(B) $e^{x^2-y^2}$ .cos 2xy

(C) $e^{x^2+y^2}$. sin 2xy

(D) $e^{x^2+y^2}$. cos 2xy

11. Let $z = x + iy\in\mathbb{C}$. Which one among the following is not an analytic function ?

(A) a polynomial in z of degree $n > 0$

(B) e^z

(C) $e^z \cos z$

(D) $e^z + \bar{z}$

12. Let C be the circle $|z| = 3$, positively oriented. Then the value of $\int_C \frac{e^{-z}}{z^2}dz$ is

(A) $2\pi i$

(B) $-2\pi i$

(C) 0

(D) 1

13. Let $w = z + \frac{1}{z}$ be Joukowski's transformation, $z \in \mathbb{C} - \{0\}$. Then, w is

(A) Conformal at all points $z \in \mathbb{C}$

(B) Not conformal only at z = 1

(C) Conformal at all points $z \in \mathbb{C}$ except at z = 1 and z = –1

(D) Not conformal anywhere

14. Suppose x, y are real and satisfy the equation $\frac{iy}{ix+1} - \frac{3y+4i}{3x+y} = 0$

Then, a possible solution (x, y) is

(A) $\left(\frac{3}{2}, -2\right)$

(B) $\left(-\frac{3}{2}, 2\right)$

(C) $\left(\frac{3}{2}, \frac{3}{2}\right)$

(D) $\left(-\frac{3}{2}, -2\right)$

15. Let V be a vector space over a field F. Which one of the following statements is true ?

(A) Any non-empty finite generating subset of V contains a basis of V

(B) V is always isomorphic to the vector space F^n for some positive integer

(C) V always has infinitely many elements

(D) V can never contain a subspace with finitely many elements

16. Let R be an integral domain. Then

(A) R can never be a field

(B) R[x] may not be an integral domain

(C) R is not a quotient ring of R[x]

(D) R is a quotient ring of R[x]

17. Let E_1 and E_2 be two finite extension fields of the field of rational numbers $\mathbb{Q}$. If degree of E_1 over $\mathbb{Q}$ is d_1 and degree of E_2 over $\mathbb{Q}$ is d_2. Then

(A) $d_1 > d_2$ implies $E_1 \supset E_2$

(B) $d_2 | d_1$ implies $E_1 \supset E_2$

(C) $E_1 \supset E_2$ implies $d_2 | d_1$

(D) $E_1 \supset E_2$ may not imply $d_2 | d_1$

18. Let G be a finite group of order n. Then which one of the following is true ?

(A) G is isomorphic to a subgroup of S_n, the symmetric group on n-symbols

(B) G can always be mapped injectively as a subgroup into the group $GL_2(\mathbb{C})$, of invertible 2 × 2 complex matrices

(C) G can always be mapped injectively as a subgroup into the group $GL_2(\mathbb{R})$, of invertible 2 × 2 real matrices

(D) G can always be mapped injectively as a subgroup into a group H of order 3^n

19. In the polynomial ring $\mathbb{Z}[x]$ over the integers which one of the following statements holds ?

(A) $\mathbb{Z}[x]$ has only finitely many maximal ideals

(B) Every non-zero prime ideal of $\mathbb{Z}[x]$ is maximal

(C) Every ideal of $\mathbb{Z}[x]$ is generated by a single element

(D) $\mathbb{Z}[x]$ has infinitely many ideals which are not generated by single element

20. Let G_1 and G_2 be two groups of order 49 and $\phi : G_1 \to G_2$ be a non-trivial homomorphism. Then

(A) ϕ is always one-one

(B) ϕ is always an isomorphism

(C) ϕ is always onto

(D) ϕ may not be one-one

21. Let ζ be a primitive 5^{th} root of unity. Then $\zeta + \zeta^2 + \zeta^3$ is equal to

(A) $\zeta^4 - 1$

(B) $-\zeta^4 - 1$

(C) ζ^4

(D) –1

22. Let p_1 and p_2 be two distinct positive odd integers and S be the set of positive integers less than or equal to p_1p_2 which are coprime to p_1p_2. Then the cardinality of S

(A) May be equal to $(p_1 - 1)(p_2 - 1)$

(B) Is always equal to $(p_1 - 1)(p_2 - 1)$

(C) May be equal to $p_1 + p_2 - 2$

(D) Is always equal to $p_1 + p_2 - 2$

23. Let A be a non-empty set of real numbers which is bounded below. Then

(A) inf A = sup(–A)

(B) inf A = – sup(–A)

(C) inf A = – sup A

(D) inf (–A) = sup A

24. $\int_0^1 \left(\log \frac{1}{x}\right)^{-1/2} dx$ is equal to

(A) $\frac{\sqrt{\pi}}{2}$

(B) $\frac{\pi}{2}$

(C) $\sqrt{\pi}$

(D) π

25. Let f be a real continuous function on a metric space X. Then the set of zeros of f is

(A) Closed in $\mathbb{R}$

(B) Closed in X

(C) Neither open nor closed in X

(D) Neither open nor closed in $\mathbb{R}$

26. $\lim_{n\to\infty} \frac{1}{n}\left(\frac{1}{1^{\sqrt{5}}} + \frac{1}{2^{\sqrt{5}}} + \ldots + \frac{1}{n^{\sqrt{5}}}\right)$ is equal to

(A) 1

(B) 0

(C) $+\infty$

(D) $\sqrt{5}$

27. Which one of the following series is divergent ?

(A) $\sum_{n=3}^{\infty} \frac{1}{(\log\log n)^{\log n}}$

(B) $\sum_{n=1}^{\infty} \frac{a^n}{n!}$, $a > 0$

(C) $\sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n}}}$

(D) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^3+1}$

28. Which one of the following is a function of bounded variation on [0, 1] ?

(A) $f(x) = \begin{cases} x\cos\frac{\pi}{2x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

(B) $f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$

(C) $f(x) = \begin{cases} x^2\cos\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

(D) $f(x) = \begin{cases} \sin\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

29. Which one of the following series is convergent ?

(A) $\sum_{n=1}^{\infty} \left(\frac{2n+1}{3n+2}\right)$

(B) $\sum_{n=1}^{\infty} \frac{(n+1)(n+2)\ldots(n+n)}{n^n}$

(C) $\sum_{n=0}^{\infty} \frac{\text{arc}\tan n}{1+n^2}$

(D) $\sum_{n=1}^{\infty} \frac{1.\,6.\,11\ldots(5n-4)}{2.\,5.\,8\ldots(3n-1)}$

30. Let S' be the unit circle in the complex plane with usual topology and $[0, 2\pi]$ be a subspace of $\mathbb{R}$ with usual topology. Then the map $f:[0, 2\pi] \to S'$ given by $f(\theta) = e^{i\theta}$ is

(A) A bijection which is continuous

(B) A homeomorphism

(C) A bijection which is not continuous

(D) A bijection whose inverse is continuous

31. With respect to discrete topology on the real line $\mathbb{R}$, which one of the following statements is false for the subset $\mathbb{Q}$ of rational numbers ?

(A) Open

(B) Closed

(C) Open and Closed

(D) Dense

32. If A is a subset of a topological space X, then which one of the following statements is false ?

(A) $\text{Bd}\,A = \overline{A} \cap \overline{(X - A)}$

(B) $\text{Int } A \cap \text{Bd } A \neq \phi$

(C) $\overline{A} = \text{Int } A \cup \text{Bd}\,A$

(D) $\text{Bd } A = \phi$ if A is both open and closed

33. The set of all limits of the sequence $x_n = \frac{1}{n}$, n = 1, 2,..., in the finite complement topology of $\mathbb{R}$ is

(A) {0}

(B) {0, 1}

(C) ϕ

(D) $\mathbb{R}$

34. Which one of the following statements is false ?

(A) Every compact Hausdorff space is normal

(B) Every regular Lindelöf space is normal

(C) Every normal space is metrizable

(D) Every well-ordered set in the order topology is normal

35. The function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x, y) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2} & , \text{ if } (x, y) \neq (0, 0) \\ 0 & , \text{ if } (x, y) = (0, 0) \end{cases}$$

(A) Is continuous at (0, 0)

(B) Is not continuous at (0, 0)

(C) Is continuous but not differentiable at (0, 0)

(D) Is continuous and differentiable at (0, 0)

36. Let $ax_1 + bx_2 = \alpha$ and $cx_1 + dx_2 = \beta$ be two linear equations with a, b, c, d, α, β all real numbers. Which one of the following statements is true ?

(A) Always there exist real numbers x_1, x_2 satisfying both equations

(B) If the rank of the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is one then there exist x_1, x_2 satisfying both the equations

(C) If the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \neq \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, then there is always x_1, x_2 satisfying both the equations

(D) If the rank of the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is 2 then there is always x_1, x_2 satisfying both the equations

37. Let T be an endomorphism of a finite dimensional vector space over a field. Which one of the following is true ?

(A) Matrix of T depends on a basis

(B) Matrix associated to T and T^2 with respect to a basis is same

(C) Matrix associated to T and T^2 with respect to a basis is always distinct

(D) Matrix of T is independent of the basis

38. Let M be a real 4×4 matrix with characteristic polynomial as a product of $x^2 + 1$ and $x^2 + 2$. Then M is similar to

(A) A diagonal matrix over $\mathbb{C}$

(B) A diagonal matrix over $\mathbb{R}$

(C) A diagonal matrix over $\mathbb{Q}$

(D) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

39. Let V be a finite dimensional complex inner product space. Which one of the following is true ?

(A) Any basis of V is orthonormal

(B) Any generating set of V contains an orthonormal basis

(C) From any basis one can obtain an orthonormal basis

(D) Every basis contains atleast two orthonormal vectors

40. Let A be a 3×3 real symmetric matrix. Which one of the following statements is true ?

(A) All eigen values of A are real

(B) All eigen values of A are always ± 1

(C) All eigen values of A are always ± 1 or 0

(D) All eigen values of A are always non- negative real numbers

41. $M_n(\mathbb{C})$ denotes the algebra of all $n \times n$ matrices over the field $\mathbb{C}$ of complex numbers. Which one of the following holds ?

(A) $M_n(\mathbb{C})$ has no zero divisors

(B) $M_n(\mathbb{C})$ is not a finite dimensional complex vector space

(C) Number of linearly independent vectors in $M_n(\mathbb{C})$ is always $< n^3$

(D) $M_n(\mathbb{C})$ is not a finite dimensional vector space over the field $\mathbb{R}$ of real numbers

42. Let $T : V \to V$ be a linear transformation of a finite dimensional vector space over $\mathbb{F}$ such that all the eigen values of T are non-zero and lie in F. Then there is a basis of V such that

(A) The matrix of T is always triangular

(B) The matrix of T is always symmetric

(C) The matrix of T is always invertible

(D) The matrix of T is not invertible

43. Let $\phi : \mathbb{R}^4 \to \mathbb{R}^2$ be defined by ϕ (w, x, y, z) = (w – x, y – z). A basis for the null space of ϕ is

(A) {(1, 0, 0, 0), (0, 1, 0, 0)}

(B) {(1, 1, 2, 2), (1, 0, 0, 0)}

(C) {(1, 1, 0, 0), (0, 0, 2, 2)}

(D) {(1, 0, 1, 0), (0, 2, 0, 2)}

44. The Jordan canonical form of a nilpotent 4 × 4 matrix A with A^3 not zero has to be

(A) $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

(B) $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

45. The class equation for the symmetric group S_3 on 3 symbols is

(A) 6 = 1 + 2 + 3

(B) 6 = 3 + 3

(C) 6 = 1 + 5

(D) 6 = 1 + 1 + 2 + 2

46. If Y is a random variable having absolutely continuous distribution function F, then what is the distribution of $-\ln F(y)$?

(A) Uniform over [0, 1]

(B) F

(C) T

(D) Standard exponential

47. If X and Y are independent Poisson random variables with mean 1, what is the conditional distribution of x + y = k, k = 0, 1, 2,...?

(A) Binomial $\left(k, \frac{1}{2}\right)$

(B) Binomial $\left(k, \frac{1}{3}\right)$

(C) Poisson (1)

(D) Geometric $\left(\frac{1}{2}\right)$

48. Let F_n be the distribution function of U^n, U having uniform distribution over [0, 1]. What does F_n converge to ?

(A) Degenerate distribution function degenerate at 1

(B) Degenerate distribution function degenerate at 0

(C) Degenerate distribution function degenerate at $\frac{1}{2}$

(D) Uniform [0, 1] distribution

49. Let (X, Y) be bivariate normal. Which one of the following is not true ?

(A) X and Y are univariate normal

(B) aX + bY is normal for all constants a and b, both not equal to zero simultaneously

(C) X and Y are dependent if Cov. (X, Y) = 0

(D) X given Y is normal

50. Let X and Y be independent, continuous random variables. Which one of the following is a necessary and sufficient condition for X + Y and X – Y to be independent ?

(A) X and Y are normally distributed

(B) X and Y have Gamma distribution

(C) X and Y have exponential distribution

(D) X and Y are uniformly distributed

51. If F_1, F_2,... is a sequence of distribution functions, which of the following is a distribution function ?

(A) $\sum_{k=1}^{\infty} \frac{1}{3^k} F_k$

(B) $\sum_{k=1}^{\infty} \frac{1}{k} F_k$

(C) $\sum_{k=1}^{\infty} \frac{1}{k^2} F_k$

(D) $\sum_{k=1}^{\infty} \frac{1}{2^k} F_k$

52. If X_1, X_2, ..., X_n are iid with pdf $f(x) = \frac{1}{2} e^{-|x-c|}$, $-\infty < x < \infty$, then what is the MLE of C ?

(A) $X_1 + ... + X_n$

(B) Median of $X_1, ..., X_n$

(C) Minimum of $X_1, ..., X_n$

(D) Maximum of $X_1, ..., X_n$

53. If $X_1, X_2, ..., X_n$ is a random sample from $F(x) = \begin{cases} 0 & \text{if } x < 0, \\ e^{-\frac{1}{x}} & \text{if } 0 \le x; \end{cases}$ what is the distribution of $\frac{1}{n}$ max. $\{X_1, ..., X_n\}$?

(A) Standard pareto

(B) Standard exponential

(C) Standard Frēchēt

(D) Standard normal

54. In the model $Y = X\beta + \varepsilon$, when does the least squares estimator of β coincides with its MLE ?

(A) When ε is normally distributed

(B) When ε has Poisson distribution

(C) When ε has t-distribution

(D) When ε has Cauchy distribution

55. Which of the following is true with reference to a finite irreducible Markov chain ?

(A) All states are necessarily persistent positive

(B) All states are necessarily persistent null

(C) All states are necessarily transient

(D) All states are aperiodic

56. Let $\{X_n, n \geq 0\}$ be a Markov chain with states 0, 1 and $p_{00} = \frac{2}{3}$, $p_{11} = \frac{1}{2}$. What is $\lim_{n\to\infty} p_{11}^{(n)}$?

(A) 0

(B) $\frac{1}{5}$

(C) $\frac{2}{5}$

(D) $\frac{3}{5}$

57. If $\{X_n, n \geq 1\}$ is a Markov chain with $P_{11}^{(n)} = P\left(X_{n+1} = 1 \middle| X_1 = 1\right) = \frac{1}{2^n}$, $n = 1, 2, \ldots,$ then classify state 1.

(A) Persistent positive

(B) Persistent null

(C) Transient

(D) Ergodic

58. Let μ be the mean of the off-spring distribution of a Galton-Watson branching process $\{X_n, n \geq 0\}$. Which of the following is true ?

(A) $\left\{\frac{X_n}{\mu^n}, n \geq 0\right\}$ is a martingale

(B) $\left\{\mu^n X_n, n \geq 0\right\}$ is a martingale

(C) $\{X_n, n \geq 0\}$ is a martingale

(D) $\left\{X_n + \mu^n, n \geq 0\right\}$ is a martingale

59. Steady state probability that the system is empty of an M/M/1 queue with arrival rate 2, service rate 1 and no waiting time, is equal to

(A) 1

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) 2

60. If X and Y are independent with $P(X \leq x) = \begin{cases} 0, & \text{if } x < 0, \\ 1 - e^{-x}, & \text{if } 0 \leq x; \end{cases}$ and $P(Y = 0) = \alpha = 1 - P(Y = 1)$, what is the moment generating function of Z = XY ?

(A) $\frac{1}{1-t}, |t| < 1$

(B) $\frac{\alpha t}{1-t}, |t| < 1$

(C) $\frac{1}{1-\alpha t}, |t| < 1$

(D) $\frac{1-\alpha t}{1-t}, |t| < 1$

61. Let $\{X_1,...,X_n\}$ be a random sample from exponential distribution with mean $\frac{1}{\theta}, \theta > 0$ and $H_0 : \theta \leq \theta_0$, $H_1 : \theta > \theta_0$. When does a UMP test for testing H_0 against H_1 reject H_0 ?

(A) $\sum_{i=1}^{n} X_i = k$, k a constant

(B) $\sum_{i=1}^{n} X_i \leq k$, k a constant

(C) $\sum_{i=1}^{n} X_i \geq k$, k a constant

(D) Such an UMP test does not exist

62. If φ is a characteristic function, which of the following is not a characteristic function ?

(A) $\frac{1}{1+\varphi(t)}, t \in \mathbb{R}$

(B) $\varphi^2(t), t \in \mathbb{R}$

(C) $\varphi^2\left(\frac{t}{2}\right), t \in \mathbb{R}$

(D) $\varphi^3\left(\frac{t}{3}\right), t \in \mathbb{R}$

63. Let X denote the number of defective items drawn randomly one-by-one from a lot. Given that E(X) = 12 and V(X) = 6, what is the distribution of X ?

(A) Binomial with parameters 25 and $\frac{1}{2}$

(B) Poisson with mean 2

(C) Geometric with parameter $\frac{1}{2}$

(D) Binomial with parameters 24 and $\frac{1}{2}$

64. Given the dispersion matrix $\Sigma = \begin{pmatrix} 1 & 4 \\ 4 & 100 \end{pmatrix}$, what is the percentage variance explained by the first principal component ?

(A) 79.4

(B) 99.2

(C) 89.3

(D) 91.2

65. Given that $\{X_1,..., X_n\}$ is a random sample from uniform $(0, \theta)$, $\theta > 0$, $M_n = \max.\{X_1,...,X_n\}$, what is the $(1 - \alpha)$ level shortest confidence interval for θ ?

(A) $\left(M_n, \frac{M_n}{\alpha^{\frac{1}{n}}}\right)$

(B) $\left(M_n, \alpha^{\frac{1}{n}} M_n\right)$

(C) $\left(M_n, M_n + \alpha^{\frac{1}{n}}\right)$

(D) $(\alpha M_n, M_n)$

66. Find the confounded interaction effects, given the following key blocks.

Replicate 1 : abc, (1), a, bc

Replicate 2 : (1), b, ac, abc

(A) AC and BC respectively

(B) AB and BC respectively

(C) BC and AB respectively

(D) BC and AC respectively

67. If $P(X_n = e^n) = \frac{1}{n} = 1 - P(X_n = 0)$, $n \geq 1$, which of the following is correct ?

(A) X_n converges to zero in r^{th} mean

(B) X_n converges to zero in probability

(C) X_n converges to 1 almost surely

(D) X_n does not converge in probability

68. Let $W = \frac{\log U}{\log U + \log(1-V)}$ where U and V are independent uniform random variables over (0, 1). What is the variance of W ?

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{1}{12}$

(D) $\frac{1}{6}$

69. Which of the following is necessary and sufficient for $X_n \xrightarrow{a.s.} 0$, given that $X_1, X_2,...$ is a sequence of independent random variables ?

(A) $\sum_{n=1}^{\infty} P(|x_n| > \varepsilon)$ is convergent for all $\varepsilon > 0$

(B) $\sum_{n=1}^{\infty} P(|x_n| > \varepsilon)$ is divergent for all $\varepsilon > 0$

(C) $\lim_{n\to\infty} P(|x_n| > \varepsilon) = 0$ for all $\varepsilon > 0$

(D) $\lim_{n\to\infty} P(|x_n| > \varepsilon) > 0$ for all $\varepsilon > 0$

70. Let $(X_1, Y_1),...,(X_n, Y_n)$ is a random sample from a continuous bivariate population and R_i = rank (X_i), S_i = rank (Y_i) when the sample values $X_1,...,X_n$ and $Y_1,..., Y_n$ are each ranked from 1 to n in increasing order of magnitude. Which of the following is not correct ?

(A) Spearman's rank correlation coefficient is $\frac{1-6\sum_{i=1}^{n}(R_i - S_i)^2}{n(n^2-1)}$

(B) When $R_i \equiv S_i$ then Spearman's rank correlation coefficient is equal to + 1

(C) Range for Spearman's rank correlation coefficient is [– 1, + 1]

(D) Range for Spearman's rank correlation coefficient is [0, + 1]

71. If Tn is a CAN estimator of θ with variance $\frac{\sigma^2}{n}$, then e^{Tn} is CAN for e^{θ} with variance

(A) $e^{-\theta} \cdot \frac{\sigma^2}{n}$ (B) $e^{-2\theta} \cdot \frac{\sigma^2}{n}$

(C) $e^{\theta} \cdot \frac{\sigma^2}{n}$ (D) $e^{2\theta} \cdot \frac{\sigma^2}{n}$

72. In a series system of k components, the system survival time is

(A) Minimum of the survival times of its components

(B) Maximum of the survival times of its components

(C) Mean survival time of its components

(D) Mode of the survival times of its components

73. Let X be a random variable with mean μ and variance σ^2. Then the Chebychev's inequality states that

(A) $P[|X-\mu| \geq k\sigma] \leq \frac{\sigma^2}{k^2}$

(B) $P[|X-\mu| \geq k\sigma] \leq \frac{1}{k^2\sigma^2}$

(C) $P[|X-\mu| \geq k\sigma] \leq \frac{1}{k^2}$

(D) $P[|X-\mu| \geq k\sigma] \leq k^2\sigma^2$

74. In the context of a multiple linear regression, multicollinearity arises when

(A) There is linear dependence among the columns of regression matrix

(B) The errors are correlated

(C) Then observations are dependent

(D) There are outliers

75. A population is divided into two strata. From the first stratum a 50% sample is drawn while the second stratum is completely enumerated. Then the allocation is optimum when population variances S_1^2 and S_2^2 of two strata are related as

(A) $S_2^2 \leq S_1^2$

(B) $4S_1^2 \leq S_2^2$

(C) $2S_2^2 \leq S_1^2$

(D) $S_2^2 > 2S_1^2$

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Space for Rough Work

ಚಿತ್ತು ಬರಹಕ್ಕಾಗಿ ಸ್ಥಳ
Space for Rough Work